Stat 145 Homework Solutions: Chapter 10

Problem 10.1

Both of these numbers are statistics, as they were computed from sample data.

Problem 10.3

The first number is a parameter because it describes the whole lot, while the second number is a statistic because it describes a sample of 100 bearings.

Problem 10.5

If the company only sold 12 policies, one lost home would result in a loss for the company (company costs would exceed total policy premiums). For thousands of policies, according to the law of large numbers the average claim should be $250.

Problem 10.7

(a) The standard deviation is

$\frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{3}} \approx 5.7735$

(b) Solve the following equation for n:

$$\begin{aligned} \frac{10}{\sqrt{n}} &= 5 \\ 10 &= 5\sqrt{n} \\ 2 &= \sqrt{n} \\ 4 &= n \end{aligned}$$

The mean of several measurements should be closer to the true value than a single measurement.

Problem 10.9

(a) Let X = score of student. The distribution of X is $N(300, 35)$.

$$\begin{aligned} P(X > 300) &= P(\frac{X - 300}{35} > \frac{300 - 300}{35}) \\ &= P(Z > 0) \\ &= 1 - .5 \\ &= .5 \end{aligned}$$

$$\begin{aligned} P(X > 335) &= P(\frac{X - 335}{35} > \frac{335 - 300}{35}) \\ &= P(Z > 1) \\ &= 1 - .8413 \\ &= .1587 \end{aligned}$$

(b) Let $\bar{X}$ = mean score of four students. The distribution of $\bar{X}$ is $N(300, \frac{35}{\sqrt{4}})$, or $N(300, 17.5)$.

$$\begin{aligned} P(\bar{X} > 300) &= P(\frac{\bar{X} - 300}{17.5} > \frac{300 - 300}{17.5}) \\ &= P(Z > 0) \\ &= 1 - .5 \\ &= .5 \end{aligned}$$

$$\begin{aligned} P(\bar{X} > 335) &= P(\frac{\bar{X} - 335}{17.5} > \frac{335 - 300}{17.5}) \\ &= P(Z > 2) \\ &= 1 - .9772 \\ &= .0228 \end{aligned}$$

Problem 10.11

(a) Let X = score of student. The distribution of X is $N(21, 4.7)$.

$$\begin{aligned} P(X \geq 23) &= P(\frac{X-21}{4.7} \geq \frac{23-21}{4.7}) \\ &= P(Z \geq 0.43) \\ &= 1 - .6664 \\ &= .3336 \end{aligned}$$

(b) Let $\bar{X}$ = mean score of 50 students. The distribution of $\bar{X}$ is $N(21, \frac{4.7}{\sqrt{50}})$, or $N(21, 0.665)$.

$$\begin{aligned} P(\bar{X} \geq 23) &= P(\frac{\bar{X}-21}{0.665} \geq \frac{23-21}{0.665}) \\ &= P(Z \geq 3) \\ &= 1 - .9987 \\ &= .0013 \end{aligned}$$

Problem 10.17

The first number is a statistic, the second number is a parameter.

Problem 10.19

If a large number of bets are placed, the gambler will receive a mean payoff of 94.7 cents per bet. However, since a $1 bet is made each time, the gambler's average loss is 5.3 cents.

Problem 10.21

Let $\bar{X}$ = average fee of 500 households. Since n is large, by the central limit theorem the distribution of $\bar{X}$ is approximately $N(28, \frac{10}{\sqrt{500}})$, or $N(28, 0.4472)$.

$$\begin{aligned} P(\bar{X} > 29) &= P(\frac{\bar{X}-28}{0.4472} > \frac{29-28}{0.4472}) \\ &= P(Z > 2.24) \\ &= 1 - .9875 \\ &= .0125 \end{aligned}$$

Problem 10.23

The mean is 40.125 and the standard deviation is $\frac{0.002}{\sqrt{4}} = 0.001$.

Problem 10.29

Let $\bar{X}$ = mean annual return over 45-year period. The distribution of $\bar{X}$ is approximately $N(13, \frac{17}{\sqrt{45}})$, or $N(13, 2.534)$.

$$\begin{aligned} P(\bar{X} > 15) &= P(\frac{\bar{X} - 13}{2.534} > \frac{15 - 13}{2.534}) \\ &= P(Z > 0.79) \\ &= 1 - .7852 \\ &= .2148 \end{aligned}$$

$$\begin{aligned} P(\bar{X} < 10) &= P(\frac{\bar{X} - 10}{2.534} < \frac{10 - 13}{2.534}) \\ &= P(Z < -1.18) \\ &= .1190 \end{aligned}$$