STAT 345 - Handout 3

BASED ON SECTIONS: 3.6 – 3.9

1. To attach the housing on a motor, a production line assembler must use an electrical hand tool to set and tighten four bolts. Suppose that the probability of setting and tightening a bolt in any 1-second time interval is $p = 0.8$. If the assembler fails in the first second, the probability of success during the second 1-second interval is 0.8, and so on.

NEG BIN

   (a) Find the probability distribution of X, the length of time until a complete housing is attached.

   $X \sim \text{NEG BIN}(r=4, p=.8)$

   $f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r, \quad x = r, r+1, \dots$

   $= \binom{x-1}{3}(.2)^{x-4}(.8)^4$

   (b) Find $P(X = 6)$.

   $Pr(X=6) = \binom{6-1}{3}(.2)^{6-4}(.8)^4$

   $= \binom{5}{3}(.2)^2(.8)^4 = 10(.04)(.4096) = .16384$

   (c) Find the mean and variance of X.

   $E(X) = \frac{r}{p} = \frac{4}{.8} = 5$

   $VAR(X) = \frac{r(1-p)}{p^2} = 1.25$

2. A manufacturer uses electrical fuses in an electrical system. The fuses are purchased in large lots and tested sequentially until the first defective fuse is observed. Assume that the lot contains 10% defective fuses.

GEOM

   $X \sim \text{GEOM}(p=.1), \quad f(x) = (1-p)^{x-1}p^1, \quad x = 1, 2, \dots$

   NOTE: $\sum_{x=1}^{N} ar^{x-1} = \frac{a(1-r^N)}{1-r}$

   (a) What is the probability that the first defective fuse will be one of the first five fuses tested?

   $Pr(X \le 5) = \sum_{x=1}^{5} Pr(X=x) = \sum_{x=1}^{5} p(1-p)^{x-1} = \sum_{x=1}^{5}(.1)(.9)^{x-1} = \frac{.1(1-.9^5)}{1-.9}$

   $= \frac{.1}{.1}(1-.9^5) = 1 - .59049 = .40951$

   (b) Find the mean, variance, and standard deviation for the number of fuses tested until the first defective fuse is observed.

   $E(X) = \frac{1}{p} = \frac{1}{.1} = 10$

   $VAR(X) = \frac{1-p}{p^2} = \frac{1-.1}{(.1)^2} = \frac{.9}{.01} = 90$

   $STD(X) = \sqrt{VAR(X)} = \sqrt{90} = 9.487$

BIN

3. Neutral particles released into an evacuated duct collide with the inner duct wall and are either scattered (reflected) with probability 0.16 or absorbed with probability 0.84.

$X \sim Bin(n, p)$, $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, \ldots, n$

(a) If four particles are released into the duct, what is the probability that all four will be absorbed by the inner duct wall?

$X \sim Bin(4, .84)$ $X = \#$ ABSORBED

$Pr(X=4) = \binom{4}{4}(.84)^4(.16)^0 = .49787$

(b) Exactly three of four?

$Pr(X=3) = \binom{4}{3}(.84)^3(.16)^1 = .37933$

(c) If 50 particles are released into the duct, what is the probability that exactly 10 will be reflected by the inner duct wall?

$X \sim Bin(50, .16)$ $X = \#$ REFLECTED

$Pr(X=10) = \binom{50}{10}(.16)^{10}(.84)^{40}$ ~~$= .10569$~~

$= (1.027 \times 10^{10})(1.0995 \times 10^{-8})(9.3576 \times 10^{-4}) = .10569$

HYPER GEOM

4. An experiment is conducted to select a suitable catalyst for the commercial production of ethylenediamine (EDA), a product used in soaps. Suppose a chemical engineer randomly selects three catalysts for testing from among a group of ten catalysts, six of which have low acidity and four of which have high acidity.

$X \sim HYPERGEOM(N=10, K=4, n=3)$, $X = \#$ HIGH ACIDIC, $f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$, $x = 0, \ldots, n$

(a) Find the probability that no highly acidic catalyst is selected.

$Pr(X=0) = \frac{\binom{4}{0}\binom{10-4}{3-0}}{\binom{10}{3}} = \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = \frac{(1)(20)}{120} = \frac{1}{6} = .1\overline{6}$

(b) Find the probability that exactly one highly acidic catalyst is selected.

$Pr(X=1) = \frac{\binom{4}{1}\binom{10-4}{3-1}}{\binom{10}{3}} = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{(4)(15)}{120} = \frac{60}{120} = \frac{1}{2} = .5$

POIS

5. A telephone operator handles, on average, five calls every 3 minutes. What is the probability that there will be no calls in the next minute? At least two calls?

$X = \#$ CALLS IN A MINUTE.

$X \sim Pois(\lambda = \frac{5}{3} = 1.6\overline{6})$, $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $x = 0, 1, 2, \ldots$

$Pr(X=0) = \frac{e^{-5/3}(5/3)^0}{0!} = \frac{e^{-5/3}(1)}{1} = .1889$

$Pr(X \geq 2) = 1 - \{Pr(X=0) + Pr(X=1)\}$

$= 1 - \left\{.1889 + \frac{e^{-5/3}(5/3)^1}{1!}\right\} = 1 - \{.1889 + .3148\}$

$= 1 - (.5037) = .4963$