

Stat 345 Solutions - Section 4.6 (2nd ed.)/3.6 (3rd ed.)

Problem 4-52/3-62

Let X = number of defective integrated circuits. Then, $X \sim \text{Bin}(n = 40, p = 0.01)$.

$$P(\text{product operates}) = P(X = 0) = \binom{40}{0} (0.01)^0 (0.99)^{40} = 0.6690$$

Problem 4-56/3-66

Let X = number of parts in the sample that require rework. then $X \sim \text{Bin}(n = 20, p = 0.01)$.

$$E(X) = np = 20(0.01) = 0.2 \text{ and } SD(X) = \sqrt{np(1-p)} = \sqrt{20(0.01)(0.99)} = 0.4450$$

Problem suspected if

$$\begin{aligned} X &> E(X) + 3SD(X) \\ X &> 0.2 + 3(0.4450) \\ X &= 1.535 \end{aligned}$$

(a)

$$\begin{aligned} P(X > 1.535) &= P(X > 1) \\ &= 1 - P(X \leq 1) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - \left[\binom{20}{0} (0.01)^0 (0.99)^{20} + \binom{20}{1} (0.01)^1 (0.99)^{19} \right] \\ &= 1 - [0.8179 + 0.1652] \\ &= 1 - 0.9831 = 0.0169 \end{aligned}$$

(b) Here, $X \sim \text{Bin}(n = 20, p = 0.04)$. Then,

$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - \left[\binom{20}{0} (0.04)^0 (0.96)^{20} + \binom{20}{1} (0.04)^1 (0.96)^{19} \right] \\ &= 1 - [0.4420 + 0.3683] \\ &= 1 - 0.8103 = 0.1897 \end{aligned}$$

(c) Let Y be the number of samples where $X > 1$. Then $Y \sim \text{Bin}(n = 5, p = 0.1897)$.

$$\begin{aligned} P(Y \geq 1) &= 1 - P(Y < 1) \\ &= 1 - P(Y = 0) \\ &= 1 - \binom{5}{0} (0.1897)^0 (0.8103)^5 \\ &= 1 - 0.3493 = 0.6507 \end{aligned}$$

Problem 4-60/3-70

(a) Let X be the number of mornings the light is green. Then $X \sim \text{Bin}(n = 5, p = 0.2)$.

$$P(X = 1) = \binom{5}{1} (0.2)^1 (0.8)^4 = 0.4096$$

(b) Let X be the number of morning the light is green. Then $X \sim \text{Bin}(n = 20, p = 0.2)$.

$$P(X = 4) = \binom{20}{4} (0.2)^4 (0.8)^{16} = 0.2182$$

(c)

$$\begin{aligned} P(X > 4) &= 1 - P(X \leq 4) \\ &= 1 - [0.0115 + 0.0576 + 0.1369 + 0.2054 + 0.2182] \\ &= 1 - 0.6296 \\ &= 0.3704 \end{aligned}$$