

Stat 345 Solutions - Section 5.4 (2nd ed.)/4.4 (3rd ed)

Problem 5-20/4-22

$$E(X) = \int_0^4 x \frac{1}{4} dx = \frac{x^2}{8} \Big|_0^4 = \frac{4^2}{8} - 0 = 2$$

$$\text{Var}(X) = E(X^2) - E(X)^2$$

$$E(X^2) = \int_0^4 x^2 \frac{1}{4} dx = \frac{x^3}{12} \Big|_0^4 = \frac{16}{3} = 5.33$$

$$\text{Var}(X) = 5.33 - 2^2 = 1.33$$

Problem 5-22/4-24

$$E(X) = \int_{-1}^1 x \frac{3}{2} x^2 dx = \int_{-1}^1 \frac{3}{2} x^3 dx = \frac{3}{2} \frac{x^4}{4} \Big|_{-1}^1 = \frac{3}{8}(1 - 1) = 0$$

$$E(X^2) = \int_{-1}^1 \frac{3}{2} x^4 dx = \frac{3}{2} \frac{x^5}{5} \Big|_{-1}^1 = \frac{3}{10}(1 - (-1)) = \frac{3}{5}$$

$$\text{Var}(X) = \frac{3}{5} - 0^2 = \frac{3}{5}$$

Problem 5-27/4-30

(a)

$$E(X) = \int_{1200}^{1210} 0.1x dx = 0.1 \frac{x^2}{2} \Big|_{1200}^{1210} = 0.05(1210^2 - 1200^2) = 1205$$

$$E(X^2) = \int_{1200}^{1210} 0.1x^2 dx = 0.1 \frac{x^3}{3} \Big|_{1200}^{1210} = \frac{0.1}{3}(1210^3 - 1200^3) = 1452033.33$$

$$\text{Var}(X) = 1452033.33 - 1205^2 = 8.33$$

$$SD(X) = \sqrt{8.33} = 2.89$$

(b)

$$P(1195 < X < 1205) = \int_{1195}^{1205} f(x) dx = \int_{1200}^{1205} 0.1 dx = 0.1(1205 - 1200) = 0.5$$