Stat 345 Solutions - Section 5.5 (2^{nd} ed.)/4.5 (3^{rd} ed)

Problem 5-32/4-37

Let X = thickness of photoresist. Then $X \sim Unif[0.2050, 0.2150]$.
(a) The cdf is

$$F(x) = \begin{cases} 0, x < 0.2050 \\ \frac{x-0.2050}{0.01}, 0.2050 \leq x < 0.2150 \\ 1, x \geq 0.2150 \end{cases}$$

(b)

$$\begin{aligned} P(X > 0.2125) &= 1 - P(X \leq 0.2125) \\ &= 1 - F(0.2125) \\ &= 1 - \frac{0.2125 - 0.2050}{0.01} \\ &= 1 - 0.75 \\ &= 0.25 \end{aligned}$$

(c) Find x such that $P(X > x) = 0.1$.

$$\begin{aligned} P(X > x) &= 1 - P(X \leq x) \\ &= 1 - F(x) \\ &= 1 - \frac{x - 0.2050}{0.01} \\ &= \frac{0.215 - x}{0.01} \end{aligned}$$

Thus, we have

$$\begin{aligned} \frac{0.215 - x}{0.01} &= 0.10 \\ x &= 0.214 \end{aligned}$$

We can check this:

$$\begin{aligned} P(X \leq 0.214) &= F(0.214) \\ &= \frac{0.214 - 0.2050}{0.01} \\ &= 0.9 \end{aligned}$$

(d)

$$E(X) = \frac{a+b}{2} = \frac{0.2050 + 0.2150}{2} = 0.21\mu m$$

$$Var(X) = \frac{(b-a)^2}{12} = \frac{(0.2150 - 0.2050)^2}{12} = 0.0000083\mu m^2$$