Stat 345 Solutions - Section 5.1 (3^{rd} ed.)

Problem 5.1

Check definition 5-1, pg. 143

(1) $f_{XY}(x, y) \geq 0$ for all x, y - yes

(2) $\sum_x \sum_y f_{XY}(x, y) = \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \frac{1}{8} = 1$

(3) $f_{XY}(x, y) = P(X = x, Y = y)$ - yes

Problem 5.2

(a) $P(X < 2.5, Y < 3) = P(X = 1, Y = 1) + P(X = 1.5, Y = 2) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

(b) $P(X < 2.5) = P(X = 1, Y = 1) + P(X = 1.5, Y = 2) + P(X = 1.5, Y = 3) = \frac{1}{4} + \frac{1}{8} + \frac{1}{4} = \frac{5}{8}$

(c) $P(Y < 3) = P(X = 1, Y = 1) + P(X = 1.5, Y = 2) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

(d) $P(X > 1.8, Y > 4.7) = P(X = 3, Y = 5) = \frac{1}{8}$

Problem 5.3

We first need to find the pmfs. Let's start with X:

$f_X(x) = \sum_y f_{XY}(x, y)$

x	1	1.5	2.5	3
$f_X(x)$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

Then,

$E(X) = \sum_x f_X(x) = (1)\frac{1}{4} + (1.5)\frac{3}{8} + (2.5)\frac{1}{4} + (3)\frac{1}{8} = 1.81$

Similarly, the pmf for Y is

y	1	2	3	4	5
$f_Y(y)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$

Then,

$E(Y) = \sum_y f_Y(y) = (1)\frac{1}{4} + (2)\frac{1}{8} + (3)\frac{1}{4} + (4)\frac{1}{4} + (5)\frac{1}{8} = 2.875$

Problem 5.4

(a) See Problem 5.3 above

(b) To find the conditional pmf, we find $P(Y = y|X = 1.5) = \frac{P(X=1.5,Y=y)}{P(X=1.5)}$ for all values of Y.

Thus we have, for example,

$$P(Y = 2|X = 1.5) = \frac{P(X = 1.5, Y = 2)}{P(X = 1.5)} = \frac{1/8}{3/8} = \frac{1}{3}$$

The pmf is then

y	1	2	3	4	5
$f_{Y\|X=1.5}(y)$	0	$\frac{1}{3}$	$\frac{2}{3}$	0	0

and we can check that this sums to 1.

(c) We have

$$P(X = 1.5|Y = 2) = \frac{P(X = 1.5, Y = 2)}{P(Y = 2)} = \frac{1/8}{1/8} = 1$$

and the rest of the probabilities are all 0.

(d)$E(Y|X = 1.5) = \sum_y yP(Y = y|X = 1.5) = (2)\frac{1}{3} + (3)\frac{2}{3} = \frac{8}{3}$ (using the distribution from part (b))

Problem 5.5

We are given that $f_{XY}(x, y) = c(x + y)$ for $x = 1, 2, 3$ and $y = 1, 2, 3$ and we need to find c. We know that the $\sum_x \sum_y f_{XY}(x, y) = 1$.

$$\begin{aligned}
\sum_x \sum_y f_{XY} x, y &= c \sum_x \sum_y (x + y) \\
&= c[\sum_x x + 1 + x + 2 + x + 3] \\
&= c[(1 + 1 + 1 + 2 + 1 + 3) + (2 + 1 + 2 + 2 + 2 + 3) + (3 + 1 + 3 + 2 + 3 + 3)] \\
&= 36c
\end{aligned}$$

Then setting $36c = 1$, we have $c = \frac{1}{36}$.

Problem 5.6

(a) $P(X = 1, Y < 4) = P(X = 1, Y = 1) + P(X = 1, Y = 2) + P(X = 1, Y = 3) = \frac{2}{36} + \frac{3}{36} + \frac{4}{36} = \frac{1}{4}$

(b) $P(X=1)$ is the same as (a), since Y must be less than 4.

(C) $P(Y=2)=P(X=1,Y=2)+P(X=2,Y=2)+P(X=3,Y=2)=\frac{3}{36}+\frac{4}{36}+\frac{5}{36}=\frac{1}{3}$

(d) $P(X<2,Y<2)=P(X=1,Y=1)=\frac{2}{36}$.