

Stat 345 Solutions - Section 5.7 (3rd edition)

Problem 5-87

(a) $E(2X + 3Y) = 2E(X) + 3E(Y) = 2(0) + 3(10) = 30$

(b) $Var(2X + 3Y) = 2^2Var(X) + 3^2Var(Y) = 4(4) + 9(9) = 97$

(c) Let $W = 2X + 3Y$. Then $W \sim N(30, 97)$. Thus,

$$\begin{aligned}P(W < 30) &= P\left(\frac{W - 30}{\sqrt{97}} < \frac{30 - 30}{\sqrt{97}}\right) \\&= P(Z < 0) \\&= 0.5\end{aligned}$$

(d)

$$\begin{aligned}P(W < 40) &= P\left(\frac{W - 30}{\sqrt{97}} < \frac{40 - 30}{\sqrt{97}}\right) \\&= P(Z < 1.02) \\&= 0.8461\end{aligned}$$

Problem 5-89

Let X_1 be the thickness of the first half and let X_2 be the thickness of the second half. Note that X_1 and X_2 are both $N(2, \sigma^2 = (0.1)^2)$ and that they are independent.

(a) Let Y be the total thickness, $Y = X_1 + X_2$. Then,

$$E(Y) = E(X_1 + X_2) = E(X_1) + E(X_2) = 2 + 2 = 4, \text{ and}$$

$$Var(Y) = Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 0.01 + 0.01 = 0.02.$$

$$\text{Thus, } SD(Y) = \sqrt{0.02} = 0.1414.$$

(b) We want to find $P(Y > 4.3)$. We know that $Y \sim N(4, 0.02)$.

$$\begin{aligned}P(Y > 4.3) &= P\left(\frac{Y - 4}{0.1414} > \frac{4.3 - 4}{0.1414}\right) \\&= P(Z > 2.12) \\&= 1 - P(Z < 2.12) \\&= 1 - 0.9830 \\&= 0.017\end{aligned}$$