Stat 345 Solutions - Section 8.2

Problem 8-19

We are given that $n = 9$, $\sigma = 2$, and $\bar{X} = 98$.

(a) Our hypotheses are

H_0: $\mu = 100$ vs. H_a: $\mu > 100$.

The test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{98-100}{2/\sqrt{9}} = -3.$$

The p-value is

$$p = P(Z > -3) = 0.999.$$

The p-value means that, if the null hypothesis is true, the probability of observing something as contrary to H_0 or more contrary (in the direction of the alternative) is 99.9%. Since it would not be unusual to observe something like 98 or larger, and since the p-value is larger than our $\alpha = 0.05$, we fail to reject the null hypothesis, and conclude that the fiber is acceptable.

Problem 8-21

We are given that $n = 10$, $\bar{X} = 1.5045$, and $\sigma = 0.01$.

(a) Our hypotheses are

$H_0 : \mu = 1.5$ vs. $H_a : \mu \neq 1.5$.

The test statistic is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{1.5-1.5045}{0.01/\sqrt{10}} = 1.42.$$

The p-value is

$$p = P(Z < -1.42) + P(Z > 1.42) = 2P(Z < -1.42) = 2(0.0778) = 0.1556.$$

Thus, if the null hypothesis is true, the probability of observing something as extreme or more extreme is about 15.56%. Since this would not be unusual and since the p-value is larger than $\alpha = 0.01$, we fail to reject H_0 and conclude that we don't have evidence against the true mean being equal to 1.5.