

CALCULUS REVIEW

INTEGRATION: POLYNOMIALS, e^x , SUBSTITUTION RULE, BY PARTS $\int x e^x dx$

DIFFERENTIATION: POLYNOMIALS, e^x , CHAIN RULE

INTEGRATION

POLYNOMIAL: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

EX $\int_0^1 x^5 = \frac{x^6}{6} \Big|_0^1 = \frac{1^6}{6} - \frac{0^6}{6} = \frac{1}{6}$

e^x :

EX $\int e^x dx = e^x + C$

SUBSTITUTION:

EX $\int_0^1 (3x+4)^5 dx$

$u = 3x+4$

↓

$du = 3 dx \Rightarrow dx = \frac{1}{3} du$

$\int_0^1 u^5 \frac{1}{3} du = \frac{1}{3} \frac{u^6}{6} \Big|_0^1 = \frac{(3x+4)^6}{18} \Big|_0^1 = \frac{7^6}{18} - \frac{4^6}{18}$

EX $\int e^{-3x} dx$

$u = -3x$

↓

$du = -3 dx \Rightarrow dx = -\frac{1}{3} du$

$\int e^u (-\frac{1}{3}) du = -\frac{1}{3} e^u + C = -\frac{1}{3} e^{-3x} + C$

EX $\int x e^{-x^2} dx$

$u = -x^2$

↓

$du = -2x dx \Rightarrow x dx = -\frac{1}{2} du$

$-\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{-x^2} + C$

CHECK: $\frac{d}{dx} (-\frac{1}{2} e^{-x^2}) = (-\frac{1}{2} e^{-x^2}) \frac{d}{dx} (-x^2) = -\frac{1}{2} e^{-x^2} (-2x) = x e^{-x^2}$

BY PARTS: $\int u dv = uv - \int v du$

EX $\int x e^{-x} dx$, PICK $u = x$ $\left\{ \begin{array}{l} dv = e^{-x} dx \\ du = 1 dx \end{array} \right.$

$v = -e^{-x}$

$\int x e^{-x} dx = -x e^{-x} - \int -e^{-x} (1) dx = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + C$

DIFFERENTIATION

Poly $\frac{d}{dx} x^n = n x^{n-1}$, $\frac{d}{dx} e^x = e^x$, $\frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$.