STAT 345 - Summer, 2005 - Practice Exam 1 Solutions
BASED ON SECTIONS: 2.1 – 2.6, 2.8, 3.1 – 3.4

1. An experiment involves counting the number of insect-damaged leaves on a plant. Find the sample space of this experiment. What type of sample space is this?

   $S = \{0, 1, 2, 3, \cdots\}$ This is a discrete sample space.

2. Let S be the sample space $S = \{s_1, s_2, s_3\}$. Let E_1 be the event $E_1 = \{s_2\}$ and E_2 be the event $E_2 = \{s_1, s_2\}$.

   (a) Find the compliment of E_1, E_1'.
   $E_1' = \{s_1, s_3\}$

   (b) Find $E_1 \cap E_2$.
   $E_1 \cap E_2 = \{s_2\}$

   (c) Find $E_1 \cup E_2$.
   $E_1 \cup E_2 = \{s_1, s_2\}$

3. Let $P(A) = 0.2$, $P(B) = 0.7$, and $P(A \cap B) = 0.1$. Find the following probabilities:

   (a) $P(A')$
   $P(A') = 1 - P(A) = 1 - 0.2 = 0.8$

   (b) $P(A \cup B)$
   $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.7 - 0.1 = 0.8$

   (c) $P(A' \cup B')$
   $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.1 = 0.9$

   (d) $P(B|A)$
   $$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.2} = 0.5$$

   (e) Are A and B independent? Justify your answer.
   A and B are not independent because of any **one** of the following:

      i. $P(A \cap B) = 0.1 \neq P(A)P(B) = (0.2)(0.7) = 0.14$ or
      ii. $P(B|A) = 0.5 \neq P(B) = 0.7$ or
      iii. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.7} = 0.1428571 \neq P(A) = 0.2$

   Note that you only need to give one of (i) to (iii) in your answer.

   (f) Are A and B mutually exclusive? Justify your answer.
   A and B are **not** mutually exclusive because $P(A \cap B) = 0.1 \neq 0$.

4. If A, B, and C are mutually exclusive events with $P(A) = 0.2$, $P(B) = 0.3$, and $P(C) = 0.4$, determine the following probabilities:

   (a) $P(A \cup B \cup C)$
   $P(A \cup B \cup C) P(A) + P(B) + P(C) = 0.2 + 0.3 + 0.4 = 0.9$

   (b) $P(A \cap B \cap C)$
   $P(A \cap B \cap C) = 0$ as they are mutually exclusive

   (c) $P[(A \cup B) \cap C)$
   $P[(A \cup B) \cap C) = 0$ because $A \cup B$ is mutually exclusive of C

5. Let $P(A) = 0.4$, $P(B|A) = 0.3$, and $P(B) = 0.2$. What is $P(A|B)$?
   $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{(0.3)(0.4)}{0.2} = 0.6$$

6. In a particular community, a vote on a school bond issue will be taken in a month. To assess the level of support for the bond issue, a sample of 200 registered voters in the community is taken. Each person is asked whether he or she supports the bond issue and whether he or she has children in school. The results of the survey are shown in the table below.

	Children in School?	
	Yes	No
Support bond issue	75	25
Do not support bond issue	25	40
Undecided	15	20

   Suppose that a person from this sample is randomly selected.

   (a) What is the probability that the person supports the bond issue?
   $P(\text{support}) = \frac{75+25}{200} = 0.5$

   (b) Give that the person has children in school, what is the probability that the person supports the bond issue?
   $P(\text{support}|\text{children in school}) = \frac{75}{115} = 0.652$

   (c) Given that the person does not have children in school, what is the probability that the person does not support the bond issue or is undecided?
   $P(\text{does not support or undecided}|\text{does not have kids in school}) = \frac{40+20}{85} = 0.706$

7. A lot of 50 electrical components contains 10 that are defective. Two components are selected at random, without replacement, from the lot. Find the following probabilities:

(a) The first component selected is defective.
$P(\text{first defective}) = \frac{10}{50} = 0.2$

(b) The second component selected is defective given that the first is defective.
$P(\text{second is defective}|\text{first is defective}) = \frac{9}{49} = 0.1837$

(c) The second component selected is defective given that the first is acceptable.
$P(\text{second is defective}|\text{first is not defective}) = \frac{10}{49} = 0.2041$

(d) The second component is defective (you don't know if the first component was defective or not).
$P(\text{second is defective}) = P(\text{second is defective}|\text{first is not defective})P(\text{first is not defective}) + P(\text{second is defective}|\text{first is defective})P(\text{first is defective}) = \frac{10}{49}\frac{40}{50} + \frac{9}{49}\frac{10}{50} = \frac{10}{50}$

8. A manufacturer uses electrical fuses in an electronic system. The fuses are purchased in large lots and tested sequentially until the first defective fuse is observed. Assume that the lot contains 10% defective fuses.

(a) What is the probability that five fuses must be tested until the first defective fuse is observed?
Let G denote a good (non-defective) fuse and let D denote a defective fuse. The probability desired is $P(GGGGD) = (0.9)(0.9)(0.9)(0.9)(0.1) = 0.06561$.

(b) What is the probability that the first defective fuse will be one of the first five fuses tested?
One of the first five fuses means it can be the first fuse, $2^{nd}, 3^{rd}, 4^{th}$ or 5^{th} fuses. Therefore the probability that it is one of the first five is $P(D) + P(GD) + P(GGD) + P(GGGD) + P(GGGGD) = 0.1 + (0.9)(0.1) + (0.9)^2(0.1) + (0.9)^3(0.1) + (0.9)^3(0.1) = 0.41$.

9. The pmf of a discrete random variable Y is given by

y	-4	0	2
$f(y)$	0.2	k	0.25

(a) What is the value of k? $k = 0.55$

(b) Find $E(Y)$. -0.3

(c) Find $Var(Y)$. 4.11

(d) Find the cdf of Y. You must define the cdf for all values of Y to receive full credit.

$$F(y) = P(Y \leq y) = \begin{cases} 0 & y < -4 \\ 0.2 & -4 \leq y < 0 \\ 0.75 & 0 \leq y < 2 \\ 1 & 2 \leq y \end{cases}$$

10. Consider the discrete random variable X with finite range $\{1, 2, 3, 4\}$ and probability mass function:

x_i	$f(x_i)$
1	0.2
2	0.2
3	0.3
4	0.3

(a) Find $P(X < 1)$. 0

(b) Find $P(X \leq 1)$. 0.2

(c) Find $P(X \leq 2)$. 0.4

(d) Find $P(X \leq 3)$. 0.7

(e) Find $P(X \leq 4)$ 1.0

(f) Find $E(X)$. 2.7

(g) Find $Var(X)$. 1.21

(h) Sketch the cumulative distribution function $F(t) = P(X \leq t)$; hint: you've already done most of the work.
See class notes.