**STAT 345 - Summer, 2005 - Practice Exam 2**

BASED ON SECTIONS: 3.5 – 3.9, 4.1 – 4.7, 4.9, 5.1, 5.5, 5.7

1. A machine that fills bottles of Coca Cola is within tolerance 98% of the time (although not necessary to complete this problem, assume that "within tolerance" means a bottle has between 11.9 and 12.1 ounces in it). Assuming that bottles are filled independently of each other, let $X$ denote the number of bottles filled until one is filled within tolerance.

   (a) What is the distribution of $X$?

   (b) Find $P(X = 1)$.

   (c) Find $P(X \leq 2)$.

   (d) Find $P(X > 2)$.

   (e) What is the expected number of bottles filled before one is filled within tolerance?

   (f) What is the expected number of bottles filled before two are filled within tolerance?

2. Let's reconsider the bottling machine. Let $X$ be the number of bottles filled within tolerance in a 6-pack of Coke.

   (a) What is the distribution of $X$?

   (b) What is the expected number of correctly filled bottles in a 6-pack?

   (c) What is the probability that all 6 bottles are *incorrectly* filled, i.e. not filled within tolerance?

   (d) What is the probability that fewer than 2 bottles are *correctly* filled within tolerance?

3. Pretend there is a gilded urn filled with 15 black balls and 5 white balls. You are to randomly draw 10 balls from the urn without replacement. Let $X$ denote the number of white balls in your draw of 10 from the urn.

   (a) What is the distribution of $X$?

   (b) What is the expected number of white balls drawn from the urn in a sample of size 10?

   (c) What is $P(X = 0)$?

   (d) What is the range of $X$?

4. The number of cars that arrive at an intersection during a specified period of time, often possess (approximately) a Poisson probability distribution. When the mean arrival rate $\lambda$ is known, the Poisson probability distribution can be used to aid a traffic engineer in the design of a traffic control system. Suppose you estimate that the mean number of arrivals per minute at the intersection is one car per minute.

   (a) What is the probability that in a given minute, the number of arrivals will equal three or more?

   (b) Can you assure the engineer that the number of arrivals will rarely exceed three per minute?

5. Let $X \sim \text{Bin}(3, 0.5)$. Sketch the cumulative distribution function $F(t) = P(X \leq t)$ for the r.v. $X$.

6. Suppose the continuous random variable $X$ has the following probability density function:

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

   (a) Find the $P(X > 3)$.

   (b) Find the $P(3 < X < 4)$.

   (c) Determine $x$ such that $P(X < x) = 0.1$.

   (d) Sketch the probability density function of $X$.

   (e) Specify completely the cumulative distribution function of $X$, $F(x) = P(X \leq x)$.

7. Suppose the random variable $X$ has a continuous uniform distribution over [-2,2].

   (a) Write down the probability density function, $f(x)$ for $X$.

   (b) Find the expected value of $X$ and the variance of $X$.

   (c) Determine $x$ such that $P(-x < X < x) = 0.8$.

   (d) Find the expected value of the random variable $e^X$.

   (e) Find the expected value of the random variable $X^2$.

8. Suppose the continuous random variable $X$ has the following probability density function:

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find the expected value of $X$ and the variance of $X$.

(b) What is the expected value and variance of $10X$?

(c) Find the $P(X > 0.5)$.

(d) Find the $P(0.25 < X < .75)$.

(e) Specify completely the cumulative distribution function of $X$, $F(x) = P(X \leq x)$.

9. Suppose that we have three independent random variables, $X_1$, $X_2$ and $X_3$. Further suppose that $E(X_1) = 2$, $Var(X_1) = 1$,$E(X_2) = 5$, $Var(X_2) = 4$,$E(X_3) = 10$, and $Var(X_3) = 3$.

(a) Find the $E(X_1 + X_2 + X_3)$.

(b) Find the $E(3X_1 + 2X_3)$.

(c) Find the $Var(X_1 + X_2 + X_3)$.

(d) Find the $Var(3X_1 + 2X_3)$.

(e) Find the $Var(4(X_2 + X_3))$.

(f) Find the $Var(2X_1 - 5X_3)$.

(g) Find the $E(2X_1 - 5X_3)$.

10. The loaves of rye-bread distributed to local stores by a certain bakery have an average length of 30 centimeters and a standard deviation of 2 centimeters. Assume that the lengths are normally distributed.

(a) What percentage of loaves are longer than 31.7 centimeters?

(b) What percentage of loaves are between 29.3 and 33.5 centimeters?

(c) What percentage of loaves are shorter than 25.5 centimeters?

(d) Suppose we need to order bags of the appropriate size for the loaves of bread. Find the length which we would expect 99 percent of the loaves to be under.

11. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with mean of 4 minutes (That is, $E(X) = \dfrac{1}{\lambda} = 4$ minutes).

(a) What is the probability that a person is served in less than 3 minutes?

(b) What is the probability that in 4 of the next 6 days (assuming that days are independent) a person is served in less than 3 minutes? (Hint: This random variable is different from that in part (a).)