Name: Key

STAT 345 - Summer, 2006: Quiz 6 (take home)
BASED ON SECTIONS: 3.6 – 3.9

For each problem below:

- Give a description of the random variable of interest.
- Find the distribution of the random variable.
- Set up, but do not evaluate, the probability of interest.

1. A personnel director selects two employees for a certain job from a group of six employees, of which one is a woman and five are men. Find the probability that the woman is selected for one of the jobs.

   $X =$ number of women selected in the sample of size $n = 2$
   $x = 0, 1$
   $X \sim$ Hypergeometric(6,1,2)
   $P(X = 1) = \frac{\binom{1}{1}\binom{5}{1}}{\binom{6}{2}} = \frac{1}{3}$

2. The probability of winning in a certain state lottery is said to be about $\frac{1}{9}$. Suppose it is exactly $\frac{1}{9}$ and that tickets are independent. What is the distribution of the number of tickets a person must purchase until they have a winning one?

   $X =$ the number of tickets a person selects until they win
   $x = 1, 2, 3, \ldots$
   $X \sim Geom(\frac{1}{9})$

3. A single bit (0 or 1) is transmitted over a noisy communications channel, it has probability 0.1 of being incorrectly transmitted. To overcome this problem the bit is transmitted 5 times. A decoder at the receiving end decides that the correct message is that carried by a majority of the received bits. Under a simple noise model, each bit is independently subject to being corrupted with the same probability, 0.1. What is the probability that the message is correctly received?

   $X =$ the number of bits transmitted incorrectly out of 5
   $x = 0, 1, 2, 3, 4, 5$
   $X \sim Bin(5, 0.1)$
   Probability that the message is received correctly $\Leftrightarrow$ prob that there are 2 or fewer errors $= P(X \leq 2) = \sum_{x=0}^{2} \binom{5}{2} .1^x .9^{5-x} = .9914$

4. Suppose that the probability of an engine malfunction during any 1-hour period is 0.02. Find the probability that a given engine will survive 2 hours.

   $X =$ number of 1-hour periods until first malfunction
   $x = 1, 2, 3, \ldots$
   $X \sim Geom(0.02)$
   $P(X = 2) = .98^1.02^1 = 0.0196$
   or $P(X > 2) = 1 - \sum_{x=1}^{2} .98^{x-1}.02^1 = 1 - 0.0396 = 0.9604$

5. A large stockpile of used pumps contains 20% that are unusable and need repair. A repairman is sent to the stockpile with 3 repair kits. He selects pumps at random and tests them one at a time. If a pump works, he goes on to the next one. If a pump doesn't work, he uses one of his repair kits on it. What is the distribution of the number of pumps he inspects until the third kit is used?

   $X =$ number of pumps inspected until find 3 unusable
   $x = 3, 4, 5, \ldots$
   $X \sim$ Negative Bin(3,0.20)

6. An industrial firm supplies 10 manufacturing plants with a certain chemical. The probability that any one firms calls in an order on a given day is 0.2, and this is the same for all 10 plants. Find the probability that on the given day, the number of plants calling in orders is at most 3.

   $X =$ number of plants that call out of 10 possible
   $x = 0, 1, 2, \ldots, 10$
   $X \sim Bin(10, 0.2)$
   $P(X \leq 3) = \sum_{x=0}^{3} \binom{10}{x}.2^x.8^{10-x} = 0.879$

7. In testing the lethal concentration of a chemical found in polluted water it is found that a certain concentration will kill 20% of the fish that are subjected to it for 24 hours. If 20 fish are placed in a tank containing this concentration of chemical, what is the probability that after 24 hours exactly 14 survive?

   $X =$ the number of fish out of 20 that die after 24 hours
   $x = 0, 1, 2, \ldots, 20$
   $X \sim Bin(20, 0.20)$
   $P(14 \text{ survive}) = P(X = 6) = \binom{20}{6}.2^6.8^{14} = 0.1091$

8. Suppose that 30% of the applicants for a certain industrial job have advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool. Find the probability that the first applicant having advanced training in programming is found on the fifth interview.

   $X =$ number interviewed until 1 has advanced training
   $x = 1, 2, 3, \ldots$
   $X \sim Geom(0.3)$
   $P(X = 5) = .7^4.3^1 = 0.072$

9. A corporations has a pool of 6 firms, 4 of which are local, from which they can purchase certain supplies. If 3 firms are randomly selected without replacement, find the probability that at least one selected firm is not local.

   $X =$ number of local firms selected in a sample of size $n = 3$ from a population of size $N = 6$ where there are $K = 4$ local firms
   $x = 1, 2, 3$
   $X \sim$ Hypergeometric(6,4,3)
   Probability that at least 1 is not local
   $= P(X \leq 2) = \frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}} + \frac{\binom{2}{1}\binom{4}{2}}{\binom{6}{3}} = 0.8$

10. The employees of a firm that manufactures insulation are being tested for indications of asbestos in their lungs. The firm is requested to send three employees who have positive indications of asbestos on to a medical center for further testing. If 40% of the employees have positive indications of asbestos in their lungs, find the probability that ten employees must be tested in order to find three positives.

    $X =$ number tested until there are 3 positive tests
    $x = 1, 2, 3, \ldots$
    $X \sim$ NegBin(3,0.40)
    $P(X = 10) = \binom{10-1}{3-1}.6^7.4^3 = 0.0645$

11. A warehouse contains 10 printing machines, 4 of which are defective. A company randomly selects five of the machines for purchase. What is the probability that all five machines are non-defective?

    $X =$ the number of defective machines out of a sample of size $n = 5$ from a population of size $N = 10$ where there are $K = 4$ defective machines in the population
    $x = 0, 1, 2, 3, 4$
    $X \sim$Hypergeometric(10,4,5)
    probability that none are defective $= P(X = 0) = \frac{\binom{4}{0}\binom{6}{5}}{\binom{10}{5}} = 0.0238$