Name: Key

STAT 345 - Summer, 2006 - Quiz 7

BASED ON SECTIONS: 3.6 – 3.9

Show all work for full credit. Note that there are 2 sides.

1. *Organic Gardening* magazine conducted a poll to determine whether consumers would prefer organically grown fruits and vegetables over those grown with fertilizers and pesticides. If the costs of the two food types were the same, 90% said they would prefer the organic food. Sixty percent said they would prefer the organic food even if they had to pay more for it. Consider the preferences of a random sample of 20 consumers. Assuming the percentages in the poll are reflective of the population, find:

   (a) The probability that at least 16 of the 20 consumers would prefer the organically grown food, if the costs were the same.

   Let X = number out of 20 that prefer organically grown food if the costs are the same $\Rightarrow X \sim Bin(20, 0.90)$.

   $$P(\text{at least } 16) = P(X \geq 16) = \sum_{x=16}^{20} \binom{20}{x} 0.90^x (0.10)^{25-x}$$
   $$= \binom{20}{16} 0.90^{16}(0.10)^4 + \binom{20}{17} 0.90^{17}(0.10)^3 + \binom{20}{18} 0.90^{18}(0.10)^2$$
   $$+ \binom{20}{19} 0.90^{19}(0.10)^1 + \binom{20}{20} 0.90^{20} = 0.9568$$

   (b) The probability that at least 16 of the 20 consumers would prefer the organically grown food, even if the costs were higher than food grown with fertilizers and pesticides.

   Let X = number out of 20 that prefer organically grown food even if the costs were higher. $\Rightarrow X \sim Bin(20, 0.6)$.

   $$P(\text{at least } 16) = P(X \geq 16) = \sum_{x=16}^{20} \binom{20}{x} 0.60^x (0.40)^{25-x}$$
   $$= \binom{20}{16} 0.60^{16}(0.40)^4 + \binom{20}{17} 0.60^{17}(0.40)^3 + \binom{20}{18} 0.60^{18}(0.40)^2$$
   $$+ \binom{20}{19} 0.60^{19}(0.40)^1 + \binom{20}{20} 0.60^{20} = 0.0510$$

2. Suppose that the number of customers that enter a bank in an hour is a Poisson random variable, and suppose that $P(X = 0) = 0.03$. Determine the mean and variance of X.

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda} = 0.03 \Rightarrow \lambda = -\ln(0.03) = 3.5$$
$$\Rightarrow E(X) = 3.5 \text{ and } Var(X) = 3.5$$

3. Assume that each of your calls to a popular radio station has a probability of 0.05 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.

   (a) What is the probability that your first call that connects is your sixth call?

   Let X = the number of calls until you connect
   $P(\text{connect}) = 0.05$
   $\Rightarrow X \sim Geom(0.05)$
   $P(\text{first call that connects is tenth}) = P(X = 6) = (0.95)^5(0.05) = 0.0387$

   (b) What is the probability that it requires more than three calls for you to connect?

   $P(\text{more than three calls until you connect}) = P(X > 3) = 1 - P(X \leq 3) = 1 - \sum_{x=1}^{3}(0.95)^{x-1}(0.05) = 0.8574$

   (c) What is the mean number of calls needed to connect?

   $E(X) = \frac{1}{p} = \frac{1}{0.05} = 20$ calls on average until you connect

$X \sim \text{Bin}(n, p)$	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\text{E}(X) = np$	$\text{Var}(X) = np(1-p)$
$X \sim \text{Geom}(p)$	$\Pr(X = x) = (1-p)^{x-1} p^1$	$\text{E}(X) = \frac{1}{p}$	$\text{Var}(X) = \frac{1-p}{p^2}$
$X \sim \text{NegBin}(r, p)$	$\Pr(X = x) = \binom{x-1}{r-1}(1-p)^{x-r} p^r$	$\text{E}(X) = \frac{r}{p}$	$\text{Var}(X) = \frac{r(1-p)}{p^2}$
$X \sim \text{HyperGeom}(N, K, n)$	$\Pr(X = x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$	$\text{E}(X) = n\frac{K}{N}$	$\text{Var}(X) = n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}$
$X \sim \text{Pois}(\lambda)$	$\Pr(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$	$\text{E}(X) = \lambda$	$\text{Var}(X) = \lambda$