

**STAT 345 - Summer, 2006 - Quiz 8**

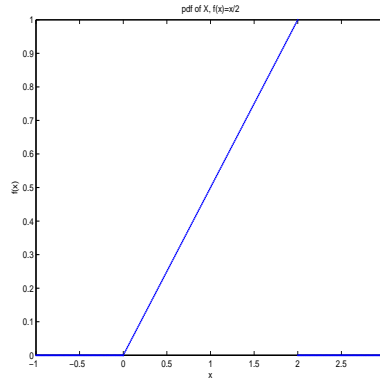
BASED ON SECTIONS: 4.2 – 4.4

Show all work for full credit.

Let the continuous random variable  $X$  have the following probability density function:

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leq x \leq 2 \\ 0 & \text{else} \end{cases}$$

1. Sketch  $f(x)$ .



2. Find  $P(X > 1)$ .

$$P(X > 1) = \int_1^2 \frac{1}{2}x dx = \frac{x^2}{4} \Big|_1^2 = 1 - 0.25 = 0.75$$

3. Find the expected value of  $X$ ,  $E(X)$ .

$$E(X) = \int_0^2 x \left(\frac{1}{2}x\right) dx = \int_0^2 \frac{1}{2}x^2 dx = \frac{1}{6}x^3 \Big|_0^2 = \frac{8}{6} = \frac{4}{3}$$

4. Find the variance of  $X$ ,  $Var(X)$ .

$$E(X^2) = \int_0^2 \frac{1}{2}x^3 dx = \frac{1}{8}x^4 \Big|_0^2 = 2$$

$$\Rightarrow Var(X) = 2 - \left(\frac{4}{3}\right)^2 = 2 - \frac{16}{9} = \frac{2}{9}$$

5. Find the c.d.f. of  $X$ ,  $F(x) = P(X \leq x)$ .

$$\int_0^x \frac{1}{2}t dt = \frac{t^2}{4} \Big|_0^x = \frac{x^2}{4}$$

$$\Rightarrow F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \leq x \leq 2 \\ 1 & 2 < x \end{cases}$$