

**STAT 345 - Summer, 2006 - Quiz 9**

BASED ON SECTIONS: 4.6, 4.9

Show all work for full credit.

1. The lifetime in hours of a lightbulb is accurately modeled as an exponential distribution with a mean of 2000 hours. If we let  $X$  be the time in hours the lightbulb lasts we have then  $X \sim \exp(0.0005)$ .

- (a) What is the probability that the lightbulb will last less than 100 hours?

$$P(X < 100) = \int_0^{100} 0.0005e^{-0.0005x} dx = -e^{-0.0005x} \Big|_0^{100} = 1 - e^{-0.05} = 0.0488$$

- (b) What is the probability that the lightbulb will last more than 1500 hours?

$$P(X > 1500) = \int_{1500}^{\infty} 0.0005e^{-0.0005x} dx = -e^{-0.0005x} \Big|_{1500}^{\infty} = e^{-0.75} = 0.4724$$

2. Let  $X \sim N(5, 4)$ ; i.e.  $X$  is normally distributed with mean  $\mu = 5$  and standard deviation  $\sigma = 2$ .

- (a) Find  $P(X < 6)$ .

$$P(X < 6) = P\left(\frac{X - \mu}{\sigma} < \frac{6 - 5}{2}\right) = P(Z < 0.5) = 0.691462$$

- (b) Find  $P(4 < X < 7)$ .

$$\begin{aligned} P(4 < X < 7) &= P\left(\frac{4 - 5}{2} < \frac{X - \mu}{\sigma} < \frac{7 - 5}{2}\right) \\ &= P(-0.5 < Z < 1) \\ &= P(Z < 1) - P(Z \leq -0.5) \\ &= 0.841345 - 0.308538 \\ &= 0.532807 \end{aligned}$$